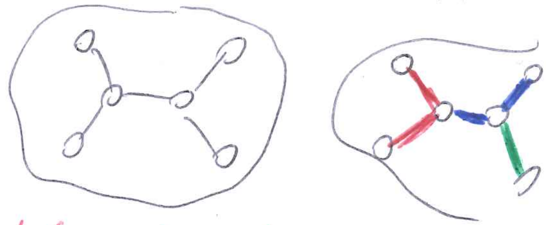
$\ell \geq 2$
G 24-edge-connected
$\delta(G)$ huge.
⇒ Decomposition into P_ℓ's and a smaller path.

Botler
Mota
Oshiro
Wakabayashi

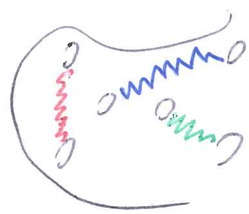
Think 24 could be 3 (best possible).
2 not good because of  + gadget.
(show it for ~~24~~ 48)

Proof:
Idea: Pick subsequent ℓ-paths along a Eulerian tour.

Many problems:
(A) ℓ^{+1} consecutive vertices ⇒ not an ℓ-path.
(B) G not Eulerian?

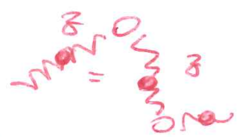
G H


Solutions:
(A) Find an Euler tour through "pre-defined paths"
⇒ basically turn G into a path-graph.

H*


$\mathcal{P}$: edge-disjoint paths of G covering E.
$H = (V, \mathcal{P})$. From H, we have H^*.
- degree of H. - H tree
- H connected. - trail.
- H eulerian.

Then, would like a tour in H where every two subsequent paths are non-conflicting
⇒ conflictless Eulerian tour. How



And of if path lengths of $H \geq \ell$ (only have to consider paths "around" a vertex).
- $(\geq \ell)$-path-graph. - (ℓ, ℓ')-path-graph.

ABS = low conflict

(B) construct but Eulerian ⇒ noise so need stronger

(B) Construct a dense "absorber", i.e. a conflictless Eulerian $(\geq \ell)$-path-graph w/ low conflicts so that it's solution extends to the "sparse" remaining "noise". How

II

Requirements ?

to create the absorber, need to construct objects
just need a lot of edge-connectivity and degree.
source of degree

ABS
- $(\geq \ell)$ path-graph
- dense
- low conflicts

+

PR
- parity repairing
- sparse

+

CR
- connectivity repairing
- sparse

ABS
- $(\geq \ell)$-path-graph.
- dense.
- low conflict
- Eulerian

conflictless Eulerian.

used in D
consumed
so $(\geq \ell)$ path graph

Basically (2) edge-co + source of degree $\Rightarrow$ PR + CR
large degree $\Rightarrow$ A.

5 key things :

(1) sparse / dense subgraphs / Fraction of graphs.

α-sparse : $\forall v, \ d_H(v) \leq \alpha \, d_G(v)$

α-dense : $\forall v, \ d_H(v) \geq \alpha \, d_G(v)$

α-fraction : $\forall v, \ d_H(v) \approx \alpha \, d_G(v)$.

(apply to path graph)

Rk : · ~~sparse~~ for path graph = low conflicts · graph = dense + sparse

(small) → ~~constant~~

· sparse + constant fraction = sparse

· dense − constant fraction = dense.

$O(\ell)$

· dense from arbitrarily large degree = ~~dense compared to~~ ℓ. large min degree

dense : max cut sparse

Th. Ellingham / Nam / Voss spanning

k-edge-co $\Rightarrow$ $1/k$ sparse spanning subgraph

some error term.

$\frac{1}{2}$-fraction exists : just induction ← w/ error term

for any α w/ finite binary expansion $\Rightarrow$ ε-fraction exists.

(just repeated iterations).

---

(2) Greedily deleting ℓ-paths $\Rightarrow$ end up w/ a graph expressable as an $(<\ell)$-path-graph w/ max degree $\ell-1$. (so sparse).

Result of Thomassen (partition into paths maximizing the lengths ...).

$\leq \ell-1$

k paths $k' \leq \ell - 2$.

$k \leq \ell - 1$ $\square$.

-1 car simple

Eulerian path-graph w/ small conflict ratio ($< \frac{1}{8}$) has a conflictless Eulerian tour. (IV)

Better: $\frac{1}{2}$ by Bill Jackson.

$\text{mult}_v(w)$: fraction of paths of v in which w appears

mult of G: max vertex mult

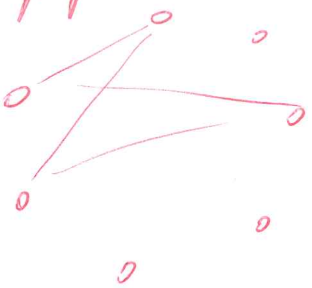
conflict ratio of P around v

conflict ration of G: max number of conflicts for a path around a vertex ($d \cdot$ mult).

Relative mult / conflict ratio.

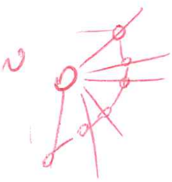
<u>Proof</u>: conflict graph around a vertex:

$\frac{1}{8}$ {



complement is <u>dense</u> ($\geq \frac{d}{2}$).

So. <u>Hamilton anticycle</u>.



of $\Rightarrow$ pairing of no conflicting path around a vertex.

$\Rightarrow$ kind of "transition"

Just go forward $\Rightarrow$ set of t closed trails (conflictless)

$t=1 \Rightarrow$ eulerian.

Otherwise

$\frac{1}{2} \leq$

$\frac{1}{8}$ conflict ratio : P_1 in conflicts w/ at ~~most~~ $\frac{1}{4}$ of pairs

P_3 same.

$\Rightarrow$ a good pair!

Just switch the...

1) PR and CR will be (maybe partially) part of ABS (V)

$\Rightarrow$ $(\geq l)$-paths

what not used from PR must be removable...

$\Rightarrow$ paths of length l or multiple of l,

and sparse so that low conflicts and not spoil of D

- for CR ~~just~~ sparse $(\geq l)$-path graph.
- for PR, system $(l, 2l)$-tree: just keep what you want to repair vertices degrees.

So construct sparse $(l, 2l)$-trees at constant cost.

2-edge-co $\Rightarrow$ subcubic $(1,2)$-tree

bounded degree $(1,k)$-tree + constant source of degree $\Rightarrow$ bounded degree $(1,k+1)$-tree

So 2-edge-co + constant source of degree $\Rightarrow$ bounded degree sparse $(1, k+1)$ tree.

+ Bip Remarks

bipartite graph 2-edge co $\Rightarrow$ $(l, 2l)$-tree w/ same property.

then $(1,k)$ $\Rightarrow$ bounded $(k, 2k)$ degree tree

Proof idea:

Just start from leaves and iteratively connect trees from the forest. Source of degrees used to extend paths too small use q-path-graph w/ large degree and low multiplicity c } → 2q-path-graph $\frac{1}{3}$ density, mult 16cq **VI**

use by constant factor.

Proof: Consider H^{ϵ}

max cut

← consider this

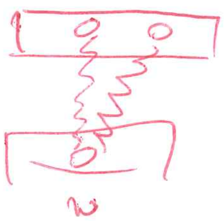
| $\frac{\alpha}{2}$ dense.
mult 2c.
large degree.

$\frac{1}{2}$ fraction | $\frac{\alpha}{4}$ dense.
mult 4c.
large degree

Cover V_1 and V_2 indep.

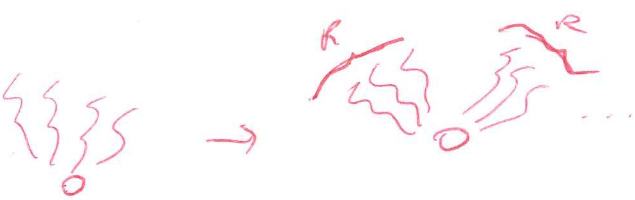
Start with V_1: turn all this into 2q paths in a dense way.

do it randomly



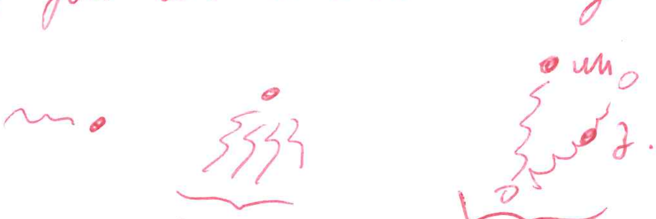
of course w can be of arbitrarily large degree (Δ)..
so high dependencies...

Idea: ~~pair into groups~~ group ~~by~~ into each conflict path and pair randomly in these.



density ok
length ok

mult? ~~Thing is that~~ fix w. In how many 2q-paths around v?



known

in one occurrence of this group...

so Chernoff to show that the ~~expe~~ # of occurences of z in $2q$ paths at a highly concentrated around the expectation

so proba z is load bounded

$\Rightarrow$ LLL because not too many dependencies (just in a group).

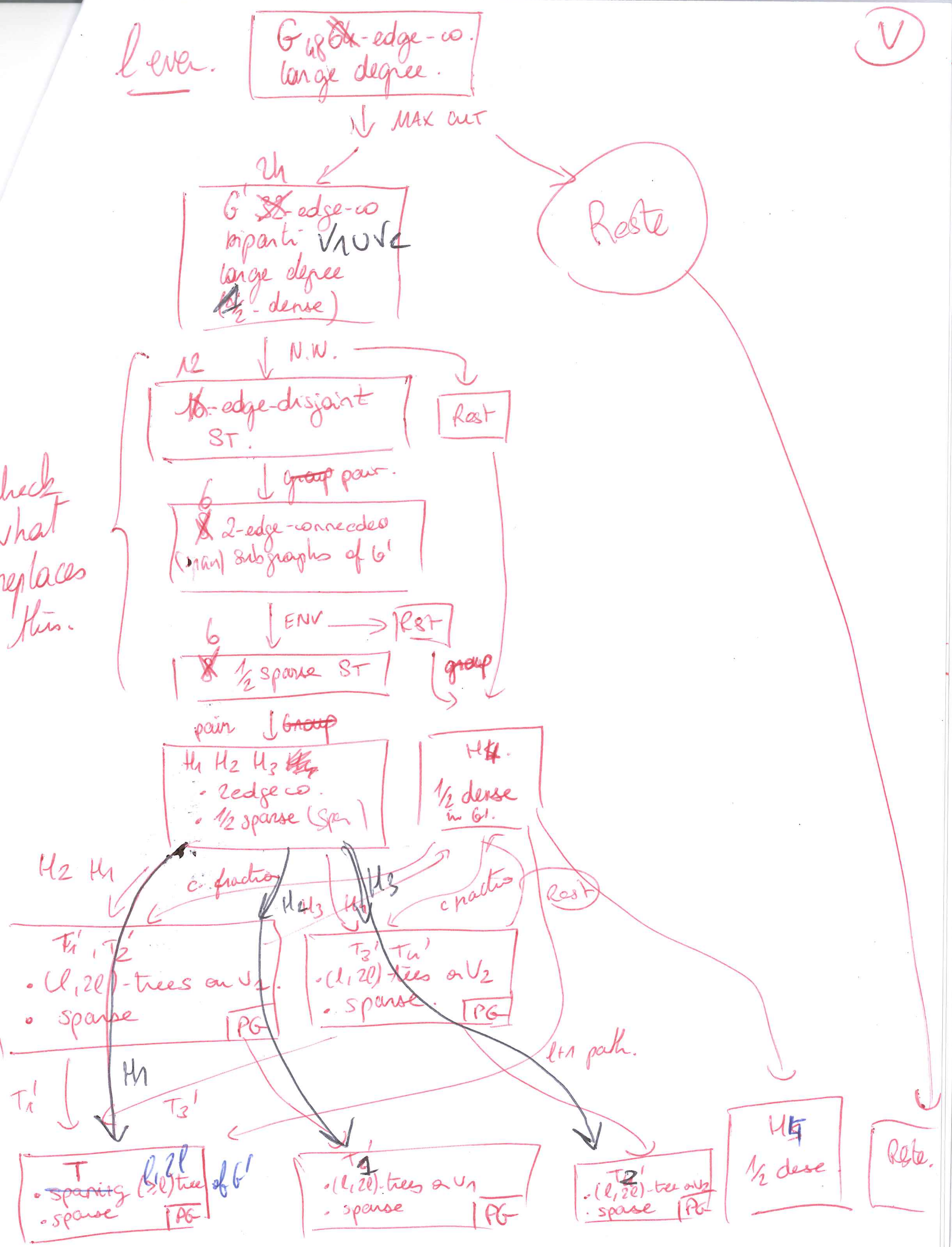
$\boxtimes$.

VII

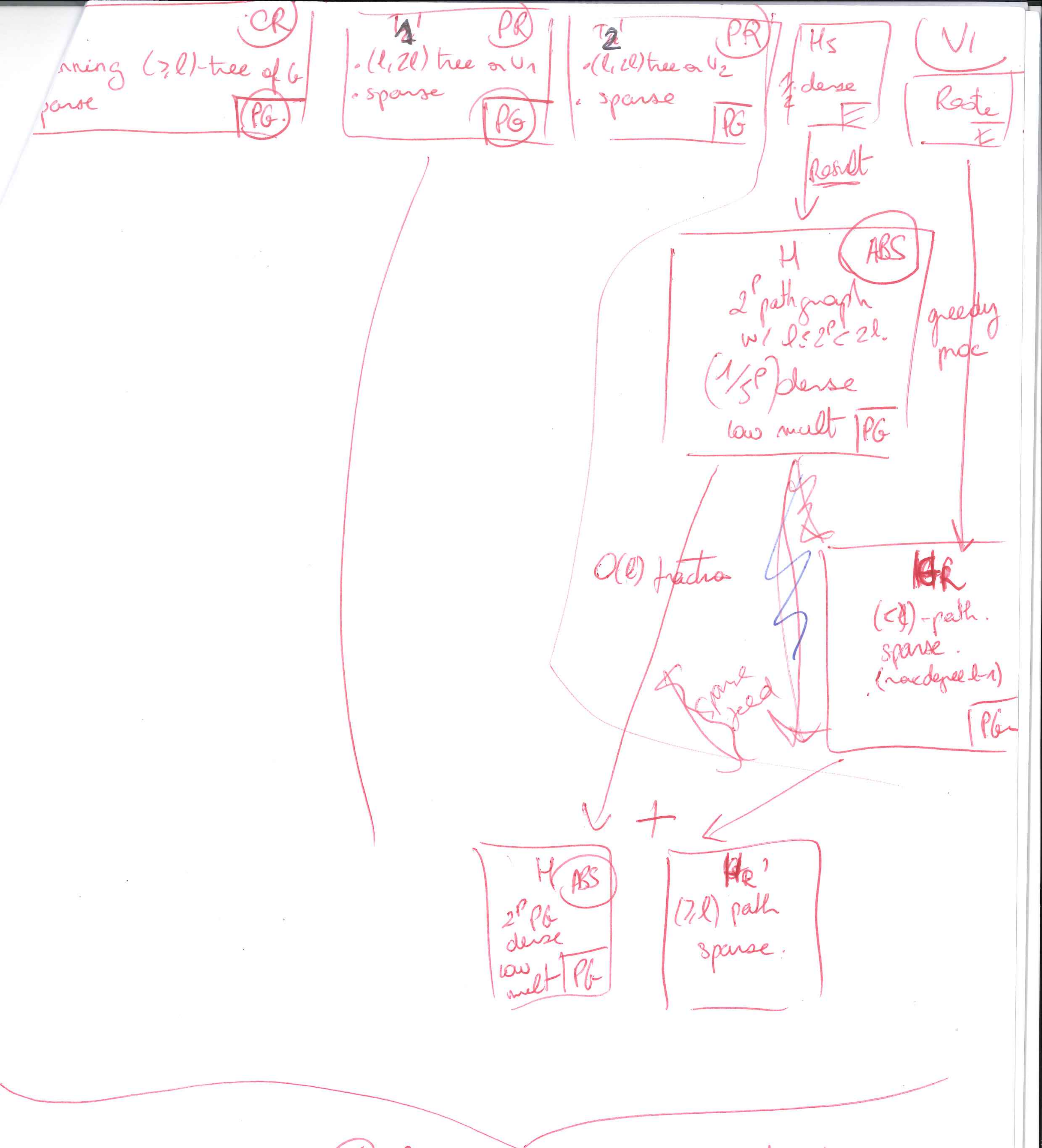
So

1-path-graph 1 dense large degree $\overset{k}{\Rightarrow}$ arbi large mult. $\frac{1}{2}$ initial	$\Rightarrow$ aim for any path length 2^p w/ high density and arb. low-density?

see V and Vi of other thing.



l even.
V
G 48 ~~6k~~-edge-co. large degree.
MAX CUT
24
G' ~~32~~-edge-co bipartite $V_1 \cup V_2$ large degree (1/2-dense)
Reste
N.W.
12
~~16~~-edge-disjoint ST.
Rest
~~group~~ pair.
6
~~8~~ 2-edge-connected (span) subgraphs of G'
check what replaces this.
ENV
Rest
6
~~8~~ 1/2 sparse ST
~~group~~
pair ~~Group~~
H_1 H_2 H_3
· 2edge co.
· 1/2 sparse (Span)
H4.
1/2 dense in G'.
H_2 H_1
c. fraction
$H_4 H_3$
H_3
c fraction
Rest
T_1', T_2'
· (l, 2l)-trees on V_1.
· sparse
PG
T_3' T_4'
· (l, 2l)-trees on V_2
· sparse.
PG
l+1 path.
H_1
T_1'
T_3'
T
· ~~sparsity~~ (l, 2l) tree of G'
· sparse
PG
T_1'
· (l, 2l)-trees on V_1
· sparse
PG
T_2'
· (l, 2l)-tree on V_2
· sparse
PG
H_5
1/2 dense
Reste.



all. ① Remove l or 2l paths from T_1' and T_2' so that evenness.

② Connectivity by T.

③ low mult cause H dense low ~~mult~~ and rest negligeable

→ DONE □